implies an unbalance of only 0.027 dB between the reference and delayed arms of the interferometer, indicating that some type of limiting may be necessary even if a SAW bandpass filter is used. Fig. 2(d) is an oscillogram of the amplitude and phase characteristics of the interferometer. The phase is very linear, running from $+90°$ to $-90°$ between each null.

In summary, the SAW interferometer described is capable of 50-dB nulls of 150-kHz periodicity over frequency bands limited by the SAW transducers used. The limiting amplifier provides the dual benefit of ultraflat frequency response using standard $[(\sin x)/x]^2$ transducers and enough gain to compensate for delay line losses. Because the nulls are extremely narrow and deep, most of the UHF band involved remains available for use in communications systems. When a satellite navigation system receiving station is unavoidably in the presence of the g-p transmission field, its highly sensitive receiver can easily be blocked so that it would be insensitive to the low level CW (with Doppler shift) received from a distant satellite. Each allocated 150-kHz-spaced g-p signal with its narrowly spaced sidebands falls well within a deep rejection notch. The frequencies of the satellite CW signals happen to fall sufficiently outside these notches to be received at an acceptable useful level.

REFERENCES

[1] P. Hartemann, "Narrow-bandwidth Rayleigh-wave filters," *Electron. Lett.*, vol. 7, pp. 674–675, Nov. 1971.

[2] P. Hartemann and O. Menager, "Rayleigh-wave discriminator," *Electron. Lett.*, vol. 8, pp. 214–215, Apr. 1972.

[3] A. J. Budreau and P. H. Carr, "Narrow-band surface wave filters at 1 GHz," in *Proc. 1972 IEEE Conf. Ultrasonics*, 1972, pp. 218–220.

[4] W. R. Smith *et al.*, "Analysis of interdigital surface wave transducers by use of an equivalent circuit model," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Microwave Acoustics*), vol. MTT-17, pp. 856–864, Nov. 1969.

[5] C. S. Hartmann, D. T. Bell, Jr., and R. C. Rosenfeld, "Impulse model design of acoustic surface-wave filters," *IEEE Trans. Sonics Ultrason.* (*Special Issue on Microwave Acoustic Signal Processing*), vol. SU-20, pp. 80–93, Apr. 1973.

An Accurate Formula for the Gamma Function

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***Abstract*—The residue calculus method of investigation of certain waveguide configurations makes use of the asymptotic properties of the gamma function. Usually the range of the variables concerned is such that this approximation is quite adequate. In a recent investigation of a very narrow waveguide junction peculiar numerical effects were traced to a condition where the variables were much too small to warrant the use of the usual asymptotic formula. A new and very simple modification extends the asymptotic form right down to zero with an error of, at most, only a few percent.**

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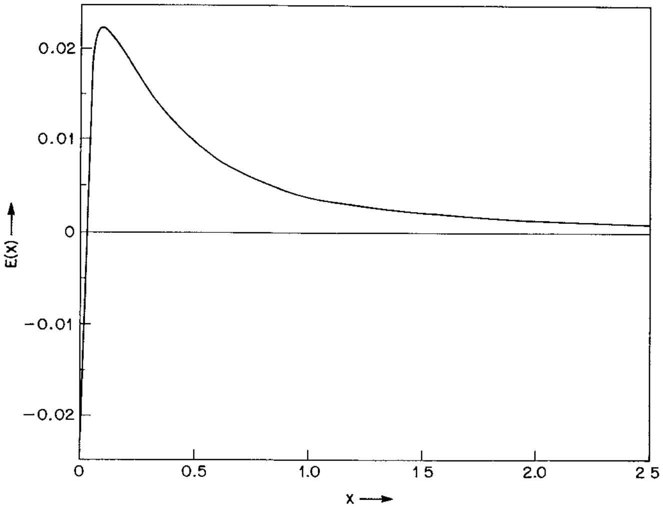

Fig. 1.

In the course of a recent investigation by the residue-calculus method of a waveguide junction with a very large dimensional ratio, it was noticed that the numerical values of some of the coefficients in the calculation were behaving quite differently from what was expected. The matter was eventually traced to an inappropriate use of the asymptotic formula for the Γ function. This is usually given in the form, valid for large x,

$$\log \Gamma(x) = (x - \tfrac{1}{2}) \log x - x + \tfrac{1}{2} \log (2\pi) + \left\{ \frac{1}{12x} - \frac{1}{360x^3} + \cdots \right\}. \quad (1)$$

In the example the values of x to be used included some close to zero, and although the correction series in $1/x$ in (1) is not usually utilized in these formulas it is clear that, even in truncated form, (1) is useless so close to the origin. The departure from the anticipated values is therefore to be expected.

In the course of working out these features an amended formula for the Γ function was found. Although it only involves a simple derivation from (1), it appears to be new, and is offered here in case it has a wider use than the particular problem that gave rise to it. It comes from incorporating the $1/12x$ term with $1/2 \log x$ (somewhat after the manner of Padé approximations), and can be written

$$\log \Gamma(x) = (x - 1) \log x - x + \tfrac{1}{2} \log (2\pi) + \tfrac{1}{2} \log (x + \tfrac{1}{6}) + E(x). \quad (2)$$

Here, $E(x)$ is a correction term which is quite small for all positive values of x, and can usually be neglected. For large x it is closely approximated by $(12x + 46/15)^{-2}$, but even for values right down to $x = 0$ it remains sufficiently small to enable the dominant terms in (2) to represent $\Gamma(x)$ to within about 2 percent. A graph of $E(x)$ from $x = 0$ to 2.5 is shown in Fig. 1.